TEST

Section - A

1. For what value of x is the matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ singular?

2. Write the principal value of $sin^{-1}\left(-\frac{1}{2}\right)$.

3. Let * be a binary operation on the set of integers I, defined by$a * b = 2a + b - 3$. Find the value of $3 * 4$.

4. Evaluate : $\int \frac{x^3-x^2+x-1}{x-1}dx]$

5. Evaluate : $\int_0^1 \frac{dx}{1+x^2}$

6. If $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$, find the value of y.

Section - B

7. A matrix A of order 3×3 is such that $|A| = 4$. Find the value of $|2A|$.

8. If $\vec{a} = 7\hat{\imath} - 2\hat{\jmath} + 3\hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + 2\hat{k}, \vec{c} = 3\hat{\imath} + 8\hat{\jmath}$, then find $\vec{a}.\left(\vec{b} \times \vec{c}\right)$ and $\left(\vec{a} \times \vec{b}\right).\vec{c}$. Also find whether they are equal.

9. Find a unit vector in the direction of $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$.

10. If the equation of a line $AB$are $\frac{3-x}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$, write the direction ratios of a line parallel to AB.

Section - C

11. Prove that :$tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}cos^{-1}x, -\frac{1}{\sqrt{2}} \le x \le 1$.

12. Let $f: R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g: R \to R$ such that $gof = fog = I_R$.

OR

A binary operation * on the set $\{0, 1, 2, 3, 4, 5\}$ is defined as $a * b = \begin{cases} a+b & \text{if} \quad a+b < 6 \\ a+b-6 & \text{if} \quad a+b \ge 6 \end{cases}$

Show that zero is the identity for this operation and each element 'a' of the set is invertible with $6 - a$, being the inverse of 'a'.

13. Using properties of determinants, solve the following for x : $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

14. Find the relationship between 'a' and 'b' so that the function 'f' defined by :$f(x) = \begin{cases} ax+1, & \text{if} \quad x \le 3 \\ bx+3, & \text{if} \quad x > 3 \end{cases}$ is continuous at $x = 3$.

If $xy = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$.

15. Prove that $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function in $\left[0, \frac{\pi}{2}\right]$.

OR

If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error on calculating its surface area.

16. If $x = \tan\left(\frac{1}{a}\log y\right)$, show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$.

17. Evaluate : $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x}$

18. Solve the differential equation :$x\,dy - y dx = \sqrt{x^2+y^2}$.

19. Using vectors, find the area of the triangle with vertices $A(1,1,2), B(2,3,5)$and $C(1,5,5)$.

20. Find the shortest distance between the following lines whose vectors equations are :

$\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k}$and $\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$.

21. Solve the differential equation : $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$

<u>SECTION - D</u>

22. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
$P(X)$	0	K	2K	2K	3K	K^2	$2k^2$	$7k^2+k$

Determine :

(i) K (ii) $P(X<3)$ (iii) $P(X>6)$ (iv) $P(0<X<3)$

Find the probability of throwing at most 2 sixes in 6 throws of a single die.

23. Using matrix method, solve the following system of equations :

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z} = 4, \frac{4}{x}-\frac{6}{y}+\frac{5}{z} = 1, \frac{6}{x}+\frac{9}{y}-\frac{20}{z} = 2; x,y,z \neq 0.$$

OR

Using elementary transformations, find the inverse of the matrix $\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$

24. Using integration, find the area of the triangular region whose sides have equations $y = 2x+1, y = 3x+1$, and $x = 4$.

25. Show that of all the rectangular inscribed in a given fixed circle, the square has the maximum area.

26. Evaluate : $\int_0^{\pi/2} 2\sin x \cos x\, tan^{-1}(\sin x)dx$

OR

Evaluate : $\int_0^{\pi/2} \frac{x\sin x\cos x}{sin^4x+cos^4x}$

27. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours craftman's time. If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectively, find the number of tennis rackets and crickets bats that the factory must manufacture to earn the maximum profit. Make it as an L.P.P. and solve graphically. During manufacturing process there may be shortage of raw material and we may try to manufacturing the racket and bats of inferior quantity. What do you suggest? Would be compromise on quality.

28. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{\imath}+2\hat{\jmath}+3\hat{k}) - 4 = 0, \vec{r}.(2\hat{\imath}+\hat{\jmath}-\hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.(5\hat{\imath}+3\hat{\jmath}-6\hat{k}) + 8 = 0$.

29. Bag I contains 3 red and 4 black balls and Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from Bag II.