TEST

Section - A

Question numbers 1 to 10 carry 1 mark each.

1. If the binary operation * on the set Z of integers is defined by $a * b = a + b - 5$, then write the identity element for the operation * in Z.
2. Write the value of $\cot(tan^{-1}a + cot^{-1}a)$.
3. If A is a square matrix such that $A^2 = A$, then write the value of $(I + A)^2 - 3A$.
4. If $x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$, write the value of x.
5. Write the value of the following determinants : $\begin{vmatrix}102 & 18 & 36\\1 & 3 & 4\\17 & 3 & 6\end{vmatrix}$.
6. If $\int \left(\frac{x-1}{x^2}\right) e^x dx = f(x)e^x + c$, then write the value of $f(x)$.
7. If $\int_0^a 3x^2 dx = 8$, write the value of 'a'.
8. Write the value of $(\hat{\imath} \times \hat{\jmath}).\hat{k} + (\hat{\jmath} \times \hat{k}).\hat{\imath}$
9. Write the value of the area of the parallelogram determined by the vector $2\hat{\imath}$and $3\hat{\jmath}$.
10. Write the direction cosines of a line parallel to z-axis.

Section - B

Question numbers 11 to 22 carry 4mark each.

11. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. What is the inverse of f?
12. Prove that : $sin^{-1}\left(\frac{63}{65}\right) = sin^{-1}\left(\frac{5}{13}\right) + cos^{-1}\left(\frac{3}{5}\right)$

    OR

    Solve for : $2\, tan^{-1}(\sin x) = tan^{-1}(2 \sec x), x \neq \frac{\pi}{2}$
13. Using properties of determinants, prove that : $\begin{vmatrix}a & a+b & a+b+c\\2a & 3a+2b & 4a+3b+2c\\3a & 6a+3b & 10a+6b+3c\end{vmatrix} = a^3$
14. If $x^m y^n = (x+y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.
15. If $y = e^{a\, cos^{-1}x}, -1 \leq x \leq 1$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

    OR

    If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1, x \neq y$, then prove that, $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$
16. Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1$, is an increasing function of x throughout its domain.

    OR

    Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

17. Evaluate : $\int x^2 \, tan^{-1} d \, dx$.

OR

Evaluate : $\int \frac{3x-1}{(x+2)^2} dx$.

18. Solve the following differential equation : $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1, x \neq 0$.

19. Solve the following differential equation : $3\, e^x \tan y \, dx + (2 - e^x) sec^2 y \, dy = 0$, given that when $x = 0, y = \frac{\pi}{4}$.

20. If $\vec{\alpha} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{\beta} = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \overrightarrow{\beta_1} + \overrightarrow{\beta_2}$, where $\overrightarrow{\beta_1}$ is parallel to $\vec{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\vec{\alpha}$.

21. Find the vector and Cartesian equations of the line passing through the point $P(1, 2, 3)$ and parallel to the planes $\vec{r}.\left(\hat{\imath} - \hat{\jmath} + 2\hat{k}\right) = 5$ and $\vec{r}.\left(3\hat{\imath} + \hat{\jmath} + \hat{k}\right) = 6$.

22. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes and hence find its mean.

Section - C

Question numbers 23 to 29 carry 6mark each.

23. Using matrices, solve the following system of equations : $x - y + z = 4; 2x + y - 3z = 0; x + y + z = 2$

OR

If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

24. Show that the altitude of the right circular cone of maximum volume that can be inscribe in a sphere of radius R is $\frac{4R}{3}$.

25. Find the area of the region in the first quadrant enclosed by x-axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

26. Evaluate : $\int_1^3 (x^2 + x)dx$ as a limit of sums.

OR

Evaluate : $\int_0^{\pi/4} \frac{cos^2 x}{cos^2 x + 4\, sin^2 x} dx$

27. Find the vector equation of the plane passing through the points $(2, 1, -1)$ and $(-1, 3, 4)$ and perpendicular to the plane $x - 2y + 4z = 10$. Also show that the plane thus obtained contains the line $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + 4\hat{k} + \lambda\left(3\hat{\imath} - 2\hat{\jmath} - 5\hat{k}\right)$.

28. A company produced soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs Rs. 10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B that costs Rs. 4. How many packets of mixes from S and T should the company purchase to honour the contract requirement and yet minimum cost? Make a LPP and solve graphically.

29. In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is girl.