TEST

SECTION – A (Objective Type Questions, 1 Mark)

1. The domain of $f(x) = sin^{-1}(-x^2)$ is

(a) $[-2, 2]$ (b) $[-1, 1]$ (c) $[0, 1]$ (d) $(0, 1)$

2. If $y = cot^{-1}\left(\frac{1}{x}\right)$, then $\frac{dy}{dx}$ is

(a) $\frac{1}{1-x^2}$ (b) $\frac{1}{1+x^4}$ (c) $\frac{1}{1+x^2}$ (d) $\frac{1}{1+x^3}$

3. The value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$ is

(a) 1 (b) -1 (c) $\frac{3}{2}$ (d) $\frac{1}{2}$

4. The value of $\int sin^2\frac{x}{2}\,dx$ is

(a) $\frac{x}{2} + \frac{\sin x}{2} + C$ (b) $\frac{x}{2} - \frac{\sin x}{2} + C$ (c) $\frac{x}{2} - \frac{\sin x}{2} - C$ (d) $\frac{x}{2} - \frac{\cos x}{2} + C$

5. If vectors $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{c} = \hat{\jmath} + \lambda\hat{k}$ are coplanar, the λ is

(a) 1 (b) -1 (c) 2 (d) -2

6. If $y = x(x-3)^2$ decreases for the value of x given by

(a) $1 < x < 3$ (b) $x < 0$ (c) $x > 0$ (d) $0 < x < \frac{3}{2}$

7. If E_1 and E_2 are two independent events such that $P(E_1) = 0.35$ and $P(E_1 \cup E_2) = 0.60$, then $P(E_2)$ is

(a) $\frac{4}{13}$ (b) $\frac{3}{13}$ (c) $\frac{5}{13}$ (d) $\frac{8}{13}$

8. $\int_0^1 xe^x dx$ is equal to

(a) 0 (b) 1 (c) e (d) e^{-1}

9. The function $f(x) = x^x$ has a stationary point at

(a) $x = e$ (b) $x = \frac{1}{e}$ (c) $x = 1$ (d) $x = \sqrt{e}$

10. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement then the probability that both drawn balls are black is

(a) $\frac{2}{7}$ (b) $\frac{1}{7}$ (c) $\frac{5}{7}$ (d) $\frac{3}{7}$

Fill in the blanks.

11. A solution of a LPP which also satisfies the non-negativity restrictions of the problem is ……………………

12. If $f(x) = |\cos x|$, then $f'\left(\frac{\pi}{4}\right)$ is equal to ……………………

OR

If $f(x) = \begin{cases} \frac{1-\sqrt{2}\sin x}{\pi - 4x}, & \text{if } x \neq \frac{\pi}{4} \\ k, & \text{if } x = \frac{\pi}{4} \end{cases}$ is continuous at $\frac{\pi}{4}$, then k is equal to ……………………

13. If A is a skew-symmetric matrix, then kA is ……………………

OR

If any two rows or columns of a determinant are identical, then its value is ……………………

14. The degree of the differential equation $\frac{d^2y}{dx^2} + e^{dy/dx} = 0$ is ……………………

15. If $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)\ \forall\ x_1, x_2\ \epsilon (a, b)$, then $f(x)$ is said to be …………………… on (a, b).

Answer the following Questions.

16. If $2A + B + X = 0$, where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$, then find the value of matrix X.

17. If $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$, then find the value of $(\vec{a} + 3\vec{b}).(2\vec{a} - \vec{b})$.

OR

Find the projection of the vector $7\hat{\imath} + \hat{\jmath} - 4\hat{k}$ on $2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$.

18. Find the function $f: R \to R$ be given by $f(x) = x^2 + 2$ and $g: R \to R$ be given by $g(x) = \frac{x}{x-1}$, then find the value of $fog(x)$.

19. Show that the function $f(x) = (x^3 - 6x^2 + 12x - 18)$ is an increasing function on R.

20. Find the equation of the tangent to the curve $y = x^2 + 4x + 1$ at the point where $x = 3$.

SECTION – B (Short Answer Type Questions, 2 Marks)

21. Find the equation of the line joining $A(1, 3)$ and $B(0, 0)$ using determinant and find k, if $D(k, 0)$ is a point such that area of $\triangle\ ABD$ is 3 sq units.

OR

Find X and Y, if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

22. A couple has two children. Find the probability that both children are males. If it is known that atleast one of the children is male.

OR

Find the probability distribution of X, the number of heads in two tosses of a coin (as a simultaneous toss of two coins)

23. Examine the continuity of the function $f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & \text{if } x \neq 4 \\ 0, & \text{if } x = 4 \end{cases}$ at $x = 4$

24. If $Y = \frac{1}{\sqrt{a^2 - x^2}}$, then find $\frac{dy}{dx}$.

25. Solve the equation for : $\cos(tan^{-1}x) = \sin\left(cot^{-1}\frac{3}{4}\right)$.

26. Find λ , when projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.

SECTION – C (Long Answer Type Questions, 4 marks)

27. Find the coordinates of point on line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{6}$, which are at a distance of 3 units from the point $(1, -2, 3)$.

OR

Show that the line $\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath})$ and $\vec{r} = (4\hat{\imath} - \hat{k}) + \mu(2\hat{\imath} + 3\hat{k})$ are coplanar. Also, find the equation of the plane containing them.

28. Find the particular solution of the differential equation $\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x\ \ (x \neq 0)$, given that $y = 0$ when $x = \frac{\pi}{2}$.

OR

Find the particular solution of the differential equation $(3xy + y^2)dx + (x^2 + xy)dy = 0$, for $x = 1, y = 1$.

29. Evaluate $\int_3^4 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{7-x}}dx$.

30. If a young man rides his motor-cycle at 25 km/h, he had to spend Rs. 2 per km on petrol with very little pollution in the air. If he rides at a faster speed of 40 km/h, the petrol cost increases to Rs. 5 per km and rate of pollution also increases.

He has Rs. 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as an LPP. Solve it graphically to find the distance to be covered with different speeds.

31. A company has two plants for manufacturing scooters. Plant I manufactures 70% of the scooters and plant II manufactures 30% of the scooters. At plant I, 30% of the scooters are maintaining pollution norms and at plant II, 90% of the scooters are maintaining pollution norms. A scooter is chosen at random and is found to b fit on pollution norms. Find the probability that it has come from plant II.

32. Consider $f: R^+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{54+5y}-3}{5}\right)$ (where, R^+ is the set of all positive real numbers).

33. Find the volume of the largest cylinder that can be inscribed in sphere of radius r.

OR

An expensive square piece of golden colour board of side 24 cm is to be made into a box without top by cutting a square from each corner and folding the flaps to forma box. What should be the side of the square piece to be cut from each corner of the board to hold maximum volume and minimize the wastage?

34. Find the equation of the plane passing through the line of intersection of planes $2x + y - z = 3, 5x - 3y + 4z + 9 = 0$ and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$.

OR

Find the distance of the point $(-2, 3, -4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

35. Using integration, find the area of the following region.

$\{(x, y): |x - 1| \le y \le \sqrt{5 - x^2}\}$

36. If $x + y + z = 0$, prove that $\begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix} = xyz \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$.